

Turbulence Model Coefficients Profiles for the Rayleigh-Taylor Instability

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Recent research has shown that initial conditions have a significant influence on the evolution of a flow towards turbulence. This important finding offers a unique opportunity for turbulence control, but also raises the question of how to properly specify initial conditions in turbulence models. We study this problem in the context of the RT instability. The RT instability is an interfacial fluid instability that leads to turbulence and turbulent mixing. It occurs when a light fluid is accelerated into a heavy fluid because of misalignment between density and pressure gradients. The RT instability plays a key role in a wide variety of natural and man-made flows ranging from supernovae to the implosion phase of ICF. Our approach is to provide a turbulence model with a predicted profile of its coefficients at the appropriate times in accordance with the initial conditions of the problem.

The Rayleigh-Taylor (RT) instability [1,2] occurs when a light fluid is accelerated into a heavy fluid. At early time, the perturbation's amplitude grows exponentially. Then, significant non-linearities appear as vorticity is generated by a baroclinic mechanism. Finally, the two fluids mix in a turbulent fashion. Recent research [3,4] has shown that initial conditions have a significant influence on the evolution of the turbulent RT instability. This characteristic offers an opportunity for "turbulence control," which may result in significant optimization for engineering applications such as inertial confinement fusion (ICF) [5]. Because traditional turbulence models used for simulating these complex problems do not capture initial condition effects, our objective is to define a rational basis for "feeding" them with initial coefficient values that reflect an initial condition influence. We propose to evolve the initial perturbation until the turbulence model hypotheses become valid, and then provide the coefficients' profiles at that time.

Our current model for predicting the evolution of the RT mixing zone is based on Goncharov's model [6] for single-mode perturbations. We compute the evolution of every existing mode of the initial perturbation spectrum, then the evolution of the bubble (spike) front is given by the envelope of the single-mode heights at all times:

$$h(t) = \max_k(h_k(t)) \quad (1)$$

where $h(t)$ is the height of the bubble (spike) front at time t and $h_k(t)$ is the height of the bubble (spike) generated by a single-mode initial perturbation at time t . The bubble (spike) front velocity is $v = dh/dt$ and the bubble (spike) front growth rate is $\alpha = h^2/4Agh$ [7].

Figure 1 illustrates how our multimode model behaves on an idealized case. Figure 1a displays an initial amplitude spectrum that could result from azimuthally averaging the 2D spectrum of an initial perturbation interface. Figure 1b shows the height of the bubble front (bold red line) as a function of time, as well as the height of a number of single modes (light black lines). Using the same color code as in Fig. 1b, Fig. 1c shows the velocity, and Fig. 1d shows the growth rate. Since we consider an ideal case, without viscosity or surface tension, the initially fastest growing mode is the mode corresponding to the largest wave number of the initial amplitude spectrum ($k = 40$ on Fig. 1a). As Figs. 1c and 1d show, the fastest growing mode leads the bubble front until $t \approx 0.8s$, so that the front grows as a single mode bubble. Between $t \approx 0.8s$ and $t \approx 1.75s$, modes corresponding to smaller and smaller wave numbers lead the bubble front. The natural pace at which the modes overtake each other at the front produces a quadratic evolution in time (implied in Fig. 1c, shown in Fig. 1d). As a result of the growth rate, Fig. 1d reaches an asymptotic value of about 0.03 between $t \approx 0.8s$ and $t \approx 1.75s$. Finally, the growth rate decays slowly after $t \approx 1.75s$. This decay is due to "missing" modes since our model does not handle mode coupling, and there is no mode generation. As the bubble front is lead by modes corresponding to smaller and smaller wave numbers, our model eventually "runs out" of modes, and the bubble front is eventually lead by the mode corresponding to the smallest existing wave number in our initial spectrum ($k = 14$ on Fig. 1a). Since the terminal velocity of a single mode is constant, its height then grows linearly, and the quadratic growth rate parameter α decays as an inverse function of time. Figure 1

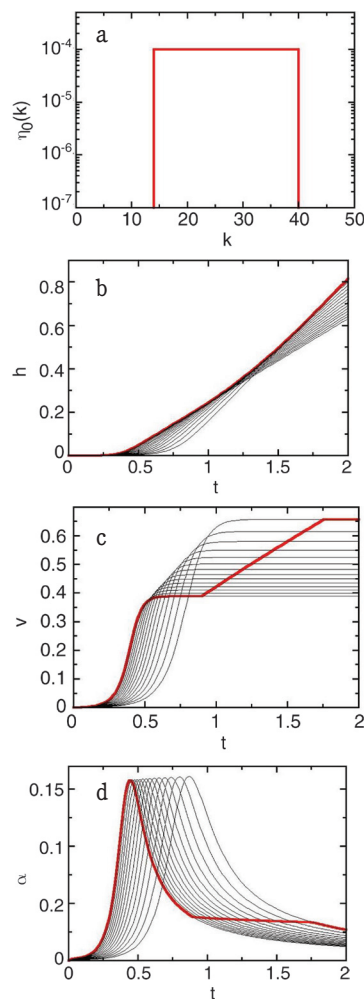


Fig. 1. Model prediction for an idealized initial amplitude spectrum. (a) Initial amplitude spectrum, (b) height, (c) velocity, and (d) growth rate as a function of time. Bold red line: bubbles' front. Light black lines: single mode bubbles.

shows that our model can reproduce the evolution of a multimode bubble (spike) front, but is limited by its inability to generate modes. Figure 2 reproduces the spectral index study made by Banerjee and Andrews [4] with our multimode model. Their 3D MILES simulations predict a late time growth rate of $\alpha \approx 0.02-0.03$ (Fig. 7b in [4]), as does our multimode without mode coupling. This figure illustrates how, for simply structured spectra, our model provides a reasonable prediction of the late time growth rate, as long as the initial amplitude spectrum is sufficiently wide.

We extract turbulence coefficients' profiles using a two-fluid model [8]. The model is based on an idealization of the mixing interface between two interpenetrating fluids. Assuming a linear distribution of the mixture fraction within the turbulent mixing zone [9], the averaged density and velocity at a given altitude z are given by:

$$\bar{\rho}(z) = f_h(z)\rho_h + f_l(z)\rho_l \quad (2)$$

$$\bar{u}_z(z) = f_h(z)u_h + f_l(z)u_l \quad (3)$$

where $f_{h/l}$ is the heavy/light fluid volume fraction. Fluctuating quantities at a given altitude are then computed using averages given by equations (2) or (3), and bulk values for the heavy fluid or the light fluid [8]. Upon substitution of the appropriate terms in the definition of a turbulence coefficient, one gets a two-fluid expression for its profile. For example, a two-fluid formulation for the mass flux, $a_z = \overline{\rho' u'_z / \bar{\rho}}$, is:

$$a_z(z) = \frac{f_h(z)f_l(z)}{f_h(z)\rho_h + f_l(z)\rho_l} (\rho_h - \rho_l)(v_s + v_b). \quad (4)$$

Since v_s , v_b , f_h and f_l are determined using the multimode model's prediction, the initial profile for parameters such as α_z can be assigned.

In conclusion, despite the lack of mode coupling, our modal model reasonably captures the growth of the turbulent mixing zone for a simply structured and sufficiently wide initial perturbation spectrum. Then profiles of turbulence model coefficients can be computed using the characteristics of the turbulence mixing zone predicted by our model and a two-fluid model. In the future, our predictions will be refined by using a multimode model that includes mode coupling, and complimentary

studies will be made on the turbulence within the turbulent mixing zone to characterize the initial time of validity for turbulence model hypotheses.

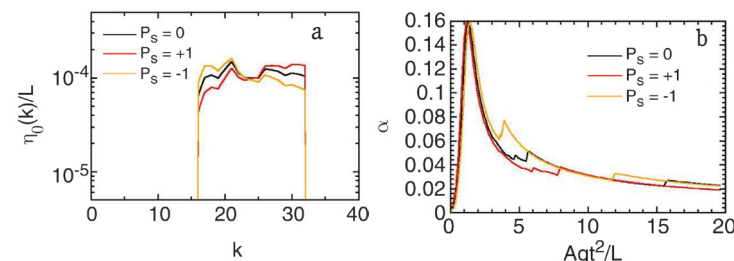


Fig. 2. Application of our model to a case found in literature. (a) Initial amplitude spectra used by Banerjee and Andrews, (b) growth rate predicted by our model, to be compared with Fig. 10 of Banerjee and Andrews [4].

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